

Amendments to Specifications:

Please replace the title of the application on [page 1] and [page 3] with the following title:

**SKILLS-BASED CONTACT CENTER SCHEDULING USING INTEGER
PROGRAMMING**

Please replace paragraph 2 on page 6 with the following amended paragraph:

U.S. Pat. No. 6,044,355 issued on March 28, 2000 to Crockett et al. describes a simulation method for developing weekly tour schedules in a contact center environment involving multiple agent groups with a plurality of skill sets, and a plurality of contact types requiring different agent skills. The method uses a scheduler and an Automatic Call Distributor (ACD) simulator. However, no working details or description of the scheduler, and how it develops a schedule is included in the patent document. It was disclosed that the method uses a “scheduler program” such as ~~TotalView™~~ available from ~~IEX Corporation~~ (U.S. Pat. No. 6,044,355, col. 6, lines 18-36). No evidence to suggest that the schedules obtained using the scheduler program ~~TotalView™~~ considered will satisfy the necessary and sufficient conditions for optimality in a workforce scheduling problem is provided or known.

Please replace paragraph 2 on page 7 with the following amended paragraph:

A primary object of the present invention is to overcome these limitations of the prior art agent scheduling methods. The present invention uses Mixed Integer Linear Programming approach (MILP) (Wolsey, 1998) to develop a mathematical model of a workforce scheduling environment. A number of researchers including Danzig (1954) proposed MILP models for workforce scheduling. Difficulties with the use of the prior art

MILP models for workforce scheduling are well documented (Nanda and Browne 1992, page 206, Holloran and Byrn, 1986, page 13). The MILP model proposed by Dantzig (1954), for example, enumerates all possible combinations of shift and tour parameters resulting in tens of trillions of decision variables, making it very inefficient and impossible to solve even in small contact center environments. To overcome these limitations of the prior art MILP agent scheduling models, the present invention formulates daily break, and days-off scheduling implicitly (i.e. not explicitly enumerating all possible combinations). Thus, the method of the present invention formulates a significantly smaller but equally powerful MILP model, and uses various extensions of it.

Please replace paragraph 1 on page 9 with the following amended paragraph:

The method of the present invention first formulates an a skills based or non-skills based agent scheduling environment involving a plurality of agent skill and work groups, and plurality of contact types as an MILP model including various contact center constraints and requirements. The formulation step of the present invention involves, first, the definitions of various decision variables, and parameters, and the formulation of an objective function. The objective function of the MILP model of the present invention determines a measure of effectiveness (or merit) for each solution. Examples of the objective function in the method include total schedule cost, total time scheduled, total paid time scheduled, total agent preference, etc. The formulation step also includes the expression of contact center constraints, and agent and skill requirements in all periods to be scheduled as mathematical equalities or inequalities. Finally, constraints that restrict the values of all decision variables to nonnegative values, and some variables to nonnegative integer values only are added to the formulation.

Please replace paragraph 1 on page 11 with the following amended paragraph:

If the best known (integer feasible) solution was changed in the RA algorithm, the B&C algorithm updates the best integer solution known for its own use, and forms new

sub-problems (nodes) by adding new constraints (Wolsey, 1998). The entire process is then repeated; the B&C algorithm chooses a new sub-problem, and solves its LP relaxation. If the LP relaxation has a feasible solution satisfying all of the constraints but some of the integrality constraints for some decision variables, the B&C algorithm transfers control to the RA algorithm which searches for an integer feasible solution through rounding and adding more agents to the schedules. The solution algorithm of the present invention terminates when all sub-problems in the B&C algorithm are terminated (for sub-problem termination conditions in the B&C algorithm are given in Wolsey, 1998). That is when the optimality condition is satisfied by an integer feasible solution to the MILP model. When this condition is satisfied, control is transferred to the schedule-reporting step.

Please replace lines 26-31 on page 13 with the following amended lines:

scheduling environment parameters provided in module (3). The present invention provides a method for the MILP model generation for a workforce scheduling environment, and an algorithm for the a near-optimal or optimal solution of the MILP model generated by the MILP model generator to develop optimal agent schedules. The method of the present invention covers both single skill non-skills based and skills based agent scheduling environments.

Please replace paragraph 4 (lines 12-18) on page 15 with the following amended paragraph:

~~A number of formulations for agent scheduling in an environment involving agents with a single skill and single contact type, and their extensions are discussed first to disclose various modeling techniques used in the MILP model generation method of the invention. A number of MILP formulations of the present invention for agent scheduling in a non-skills based (i.e. single skill) environment and their extensions are discussed first. The MILP model generation method of the present invention for skills based agent scheduling environments involving agents with different skills serving multiple contact types is~~

disclosed after this discussion. The method of the invention for optimal solution of the MILP model is disclosed thereafter.

Please add the following new paragraph after paragraph 4 (lines 12-18) on page 15:

To facilitate the description of the MILP model and its extensions used by the present invention in this patent document, any variable or set defined with a subscript is shown as flat text when the variable or set itself is a subscript of another term. For instance, a set defined as B_{1kih} is typed as B_{1kih} in $\sum_{t \in B_{1kih}}$ where it is a subscript of the summation sign Σ . Moreover, symbols “ \in ” and “ ϵ ” are both used to imply membership in a set or indicate an element of a set.

Please replace lines 25-26 on page 15 with the following amended lines:

To facilitate the presentation, this discussion assumes that all agents have the same skill set (i.e. not a skills based scheduling environment). In addition, the following

Please replace lines 22-26 on page 16 with the following amended lines:

Let K be the set of all distinct tour types available to assign agents. Each tour type k has a predetermined set of start times given in I_k . Let a_{kith} be equal to one if period t on day h is in the tour shift span (that is, a work, break, or days-off period) of tour k with a daily start time of i , and zero otherwise. Suppose that the maximum number of agents available to assign to tour k is limited to D_k^{\max} . Let X_{ki} be the number of agents assigned

Please replace lines 6-7 on page 20 with the following amended lines:

Demand constraint Constraint (2) is then formulated by adding O_{ht} for overstaffing and S_{ht} for shortages to its left side as follows:

Please replace lines 20-22 and equation (11a) on page 20 with the following amended lines and equation:

If a certain amount of overstaffing is needed for other tasks (e.g. to answer email or fax based contacts), it can be ensured by restricting the overstaffing to be greater than or equal to the desired level f_{ht} .

$$[[\sum_{t \in T_h} O_{ht} \geq f_h,]] \quad \sum_{t \in T_h} O_{ht} \geq f_{ht}, \quad t \in T_h, h = 1, \dots, 7, \quad (11a)$$

Please replace equation (11b) on page 20 with the following amended equation where d_h is corrected as d_{ht} :

$$[[\sum_{t \in T_h} S_{ht} \geq d_h,]] \quad \sum_{t \in T_h} S_{ht} \geq d_{ht}, \quad t \in T_h, h = 1, \dots, 7, \quad (11b)$$

Please replace lines 1-2 on page 21 with the following amended lines where d_h is corrected as d_{ht} :

where d_{ht} is the maximum understaffing tolerated on day h . Constraints (11a) and (11b) can easily be further modified to impose limits for a specific period(s) on day h .

Please replace equations (12a-12c) and (13a-13c) on page 22 with the following amended equations in which “=” is replaced by “≤” (please note – in the case of the greater than or equal to sign, “≤”, underlining does not work in Word):

$$[[\sum_h U_{kiht} = q_k SU_{kit},]] \quad \underline{\sum_h U_{kiht}} \leq \underline{q_k SU_{kit}}, \quad , i \in I_k, t \in D1_{ki}, \quad (12a)$$

$$[[\sum_h W_{kiht} = q_k SW_{kit},]] \quad \underline{\sum_h W_{kiht}} \leq \underline{q_k SW_{kit}}, \quad , i \in I_k, t \in D2_{ki}, \quad (12b)$$

$$[[\sum_h V_{kiht} = q_k SV_{kit},]] \quad \underline{\sum_h V_{kiht}} \leq \underline{q_k SV_{kit}}, \quad , i \in I_k, t \in D3_{ki}, \quad (12c)$$

$$\sum_{t \in D1_{ki}} SU_{kit} = 1, \quad , i \in I_k, k \in K, \quad (13a)$$

$$\sum_{t \in D2_{ki}} SW_{kit} = 1, \quad , i \in I_k, k \in K, \quad (13b)$$

$$\sum_{t \in D3_{ki}} SV_{kit} = 1, \quad , i \in I_k, k \in K, \quad (13c)$$

Please replace lines 11-16 on page 22 with the following amended lines:

where $D1_{ki}$, $D2_{ki}$, and $D3_{ki}$ are the sets containing the first consistent (across the days of a week) daily first relief, lunch, and second break times in $B1_{kih}$, $B2_{kih}$, and $B3_{kih}$, respectively, q_k is the number of work days tour k requires is a very large number, SU_{kit} , SW_{kit} , and SV_{kit} are binary variables taking a value of either zero or one to indicate whether period t is selected as the consistent daily first relief, lunch, or second relief break time, respectively.

Please replace paragraph 1 on page 27 with the following amended paragraph:

To facilitate the discussion and notation, the scheduling environment described before the disclosure of MILP model (1-8) is considered again. The present invention also extends the MILP model to tours not requiring consistent daily shift start times and

same daily shift length requirements. For these tours, both the shift start time as well as shift duration for an agent may vary from one day to another.

Please replace paragraph 2 on page 27 with the following amended paragraph:

Assume that the scheduling environment involves only tours not requiring consistent daily start time and shift length. Let QK be the set of all such tour types. Assume that agents assigned to tour type k can start at any one of a set of predetermined start times ~~for day h~~ given in QI_{kh} . Let the daily shift lengths allowed for tour type k be F_k . F_k may also change from one day to another. Note that tour k has a minimum weekly work limit specified by the shortest shift in F_k , and a maximum weekly work limit specified by the longest shift in F_k . The daily shift start times should not allow for back-to-back shift schedules for agents without allowing enough time for rest. Thus, the latest start time for a tour plus the length of the longest daily shift type are not allowed to exceed 24 hours minus the desired minimum rest period between consecutive daily shifts. To prevent this for a tour type, the present invention creates separate tour types each with start times allowing a minimum required time between the latest possible end of a shift on one day and the earliest possible start of a shift on the following day.

Please replace lines 12-14 on page 28 with the following amended lines:

start period, respectively. Let a_{knih} be equal to one if period t on day h is in the shift span (that is, a work or a break/~~days-off~~ period) of agents assigned to tour k who have the start time of i and shift length n on day h , and zero otherwise. A work pattern in this

Please replace constraint (41) on page 29 with the following amended lines:

$QX_{knhi}, Q_{kl}, QU_{knih}, QW_{knih}, \text{ and } QV_{knih} \geq 0$ and integer for all k, n, i, h and t . (41)

Please replace paragraph 2 on page 30 with the following amended paragraph:

Objective function (34) can be easily reformulated to minimize the total amount of agent time scheduled (or paid agent time) ~~scheduled~~ by setting C_{kl} equal to ~~scheduled time~~ (or paid agent time) per agent assigned to tour k and work pattern l, $l \in QL_k$, and $k \in QK$. Similarly, if agents rate their most preferred, second most preferred, etc., tours using ~~an RAting a rating~~ system, these ratings for various tours and work patterns can be aggregated and used in the objective value to maximize agent preference in scheduling tours and work patterns. ~~Note that in this case, in In~~ order to not to schedule redundant tours and increase agent costs unnecessarily, rating system should assign the lowest value to the most preferred tour, and next value higher to the second most preferred tour, etc. Consequently, the objective function will remain as of minimization type to the maximize agent preference.

Please replace last paragraph on page 31 with the following amended paragraph (subscript t is added to f_{ht}):

If a certain amount of overstaffing is needed for other tasks (e.g. to answer email or fax based contacts), it can be assured by restricting the overstaffing to be greater than or equal to the desired level f_{ht} .

Please replace constraint (45) on page 32 with the following amended constraint (46) (subscript t is added to f_{ht}):

$$[[\sum_{t \in QT_h} O_{ht} \geq f_{ht},]] \sum_{t \in QT_h} O_{ht} > f_{ht}, \quad t \in T_h, h = 1, \dots, 7, \quad (45)$$

Please replace constraint (46) and lines 9-10 on page 32 with the following amended constraint (46) (subscript t is added to d_{ht}) and lines 9-10:

$$[(\sum_{t \in QTh} S_{ht} \geq d_{ht})] \sum_{t \in QTh} S_{ht} > d_{ht}, \quad t \in T_h, h = 1, \dots, 7, \quad (46)$$

where d_{ht} is the minimum overtime desired on day h. Note that (45) and (46) can easily modified to impose limits for a specific period(s) on day h.

Please replace paragraph 1 on page 33 with the following amended paragraph:

If working hours are shorter on certain days for a tour or a contact center, the present invention extends the MILP model by excluding the daily shift variables that has a late start time resulting an end time after the early closure time for that tour or center. Note that break Break variables as well as break constraints (34-41) associated with these shift variables are excluded.

Please replace paragraph 2 and constraint (47) on page 34 with the following amended paragraph and constraint (47):

Another way that the present invention extends the MILP model is to merge formulations (1-8) and (34-41) for this type of scheduling environments to obtain optimal schedules. To distinguish cost parameters, Cx_{ki}^j is introduced as the cost of assigning an agent to X_{ki} . Using the definitions of the parameters and variables given for formulations (1-8) and (34-41), merging the objective functions (1) and (34) and demand constraints (2) and (35), and including the necessary break and days-off constraints from formulations (1-8) and (34-41), the present invention extends the MILP model for a scheduling environment involving both consistent and non-consistent shift start times and shift lengths as follows:

$$\begin{aligned}
 \text{Minimize} \quad & [[\sum_{k \in K} \sum_{i \in I_k} C_{ki} X_{ki}]] \sum_{k \in K} \sum_{i \in I_k} C_{ki} X_{ki} + \sum_{k \in QK} \sum_{l \in QL_k} C_{kl} Q_{kl} \\
 & + \sum_{k \in QK} \sum_{n \in F_k} \sum_h \sum_{i \in QI_k} c_{knih} QX_{knih} \quad (47)
 \end{aligned}$$

Please replace constraint (48) starting on page 35 and continuing to page 36 with the following amended constraint (48) (with Qik corrected as QIk in the third line and first term of the constraint):

$$\begin{aligned}
 & \sum_{k \in K} \sum_{i \in I_k} a_{kiht} X_{ki} - \sum_{k \in K} \sum_{i \in I_k} a_{kiht} (Y_{kih} + Y_{ki(h-1)}) - \sum_{k \in K} \sum_{i \in T1kht} U_{kiht} \\
 & - \sum_{k \in K} \sum_{i \in T2kht} (W_{kiht} + W_{kih(t-1)}) - \sum_{k \in K} \sum_{i \in T3kht} V_{kiht} \\
 & [[+ \sum_{k \in QK} \sum_{n \in F_k} \sum_{i \in QI_k} a_{knih} QX_{knih}]] + \sum_{k \in QK} \sum_{n \in F_k} \sum_{i \in QI_k} a_{knih} QX_{knih} - \sum_{k \in QK} \sum_{n \in F_k} \\
 & \sum_{i \in QT1knht} QU_{knih} - \sum_{k \in QK} \sum_{n \in F_k} \sum_{i \in QT2knht} (QW_{knih} + QW_{knih(t-1)}) \\
 & - \sum_{k \in QK} \sum_{n \in F_k} \sum_{i \in QT3knht} QV_{knih} \geq b_{ht} \\
 & t \in T_h, h = 1, \dots, 7 \quad (48)
 \end{aligned}$$

Please replace constraint (59) starting on page 35 with the following amended constraint (59) ("n" added between k and i):

$$X_{ki}, Y_{kih}, QX_{knih}, Q_{kl}, U_{kiht}, W_{kiht}, and V_{kiht}, QU_{knih}, QW_{knih}, and QV_{knih} \geq 0 and integer for all k, n, i, h, and t, \quad (59)$$

Please replace lines 1-9 on page 36 with the following amended lines:

where objective function (47) combines objectives functions (1) and (34), constraint (48) combined combines agent, break, and days-off variables specifying agent availability in a period from constraints (2) and (35), constraints (49-51) are the break balance constraints for the tours requiring consistent daily shift start times and shift lengths, constraint (52) ensures that sufficient number of days-off are scheduled for the agents assigned to tours requiring consistent daily shift start times and shift lengths, constraints (53-55) ensures ensure that sufficient number of first, lunch, and second relief breaks are scheduled for the agents assigned to ~~tour~~ tours not requiring consistent daily shift start times and shift lengths, constraint (56) ensures that the number of shifts scheduled on a given day is equal to the number agents who are assigned to work on that day by the work patterns scheduled, constraints (57) and (58) ensure that the numbers of agents assigned to various tours requiring consistent or non-consistent daily shift start times and shift lengths don't exceed the limits on the agent availability for these tours. Finally, constraint (59) restricts the tour, days-off, daily shift, work pattern, and break variables to nonnegative integers to avoid negative values and/or fractional agent schedules.

Please replace paragraph 1 on page 37 with the following amended paragraph:

sets. Let R be the set of different contact types with known agent requirements in each period t and day h . Let the number of agents with qualified skills for contact type $r \in R$ required in period t on day h be b_{ht}^r . Suppose now that there are a number of agent skill groups $j \in J$, each with a unique set of skills. Let the set of contact types that agent skills group j is qualified to provide service be N_j , $j \in J$. Agent requirements should reflect the multi-skill efficiency expected in a skills-based routing environment due to agents with multiple skills and, depending on the scheduling environment, may be specified for skills groups rather than contact types. A contact type may be served by multiple agent groups. Let the set of agent skill skills groups that can service contact type r be M^r .

Please replace constraint (61) starting on page 37 with the following amended constraint (61) (with “+” in front of the 5th term replaced by “-“):

$$\begin{aligned}
 & \sum_{k \in QKj} \sum_{neFk} \sum_{i \in QIk} a_{knih} QX^j_{knih} - \sum_{k \in QKj} \sum_{neFk} \sum_{i \in QT1knht} QU^j_{knih} \\
 & - \sum_{k \in QKj} \sum_{neFk} \sum_{i \in QT2knht} (QW^j_{knih} + QW^j_{knih(t-1)}) \\
 & - \sum_{k \in QKj} \sum_{neFk} \sum_{i \in QT3knht} QV^j_{knih} [[+ \sum_{r \in Nj} G^{jr}_{ht}]] - \sum_{r \in Nj} G^{jr}_{ht} = 0, \\
 & , j \in J, t \in T_h, h = 1, \dots, 7, \quad (61)
 \end{aligned}$$

Please replace paragraph 1 on page 39 with the following amended paragraph:

agents who are scheduled to be on a relief or lunch break during this period. Thus, the difference is equal to the number of agents who are scheduled and working (available) during period t on day h. Finally, the fifth term is the sum the numbers of agents allocated to various contact types in N_j.

Please replace constraint (63) starting on page 39 with the following amended constraint (63) (with “+” in front of the 2nd term replaced by “-“):

$$\begin{aligned}
 & f^j_{ht}(X, Y, Z, U, W, V, QX, Q, QU, QW, QV) [[+, + \sum_{r \in Nj} G^{jr}_{ht}]] - \sum_{r \in Nj} G^{jr}_{ht} = 0, \\
 & , j = 1, \dots, n, t \in T_h, h = 1, \dots, 7, \quad (63)
 \end{aligned}$$

Please replace lines 14-21 on page 39 with the following amended lines:

where e^{jr} is the relative efficiency of an agent from agent group j in serving contact type r with respect to an agent whose primary skill (highest proficiency level) is serving contact type r, e^{jr} ∈ [0, 1]. For instance, if e^{jr} = 1, agent group j is equally efficient, and if

$e^{jr} = 0.75$, agents in group j are 75% efficient with respect to an agent whose primary skill is serving contact type r. Because of call blending (mixing the calls routed to an agent in a planning period), values of G^{jr}_{ht} variables do not have to be integer. But their sums in each period t, day h, and for agent group j will be integers since the variables in $f_{ht}(.)$ in constraint (63) are restricted to integer values.

Please replace lines 26-29 on page 39 with the following amended lines:

groups, remain the same since the agent allocation does not affect the break and days-off scheduling constraints. Objective functions (14) and (34) are merged and modified to include potential wage differentials for different agent groups. The MILP model of the present invention is now disclosed below.

Please replace objective function (65) starting on page 40 with the following amended objective function (65) (" $\sum_{j \in J}$ " is added and " C_k^j " is changed to " Cx_k^j "):

$$\begin{aligned} \text{Minimize } & \sum_{j \in J} \sum_{i \in I_k} \sum_{k \in K_j} Cx_{ki}^j X_{ki} + \sum_{j \in J} \sum_{l \in QL_k} \sum_{k \in QK_j} C_{kl}^j Q_{kl} \\ & + \sum_{j \in J} \sum_{k \in QK_j} \sum_{n \in F_k} \sum_h \sum_{i \in QI_k} c_{kni}^j QX_{knhi}^j \quad (65) \end{aligned}$$

Please replace constraint (67) on page 40 with the following amended constraint (67) (with "+" in front of the 2nd term replaced by "-"):

$$\begin{aligned} f_{ht}^j(X, Y, Z, U, W, V, QX, Q, QU, QW, QV) \\ [[(+ \sum_{r \in N_j} G_{ht}^{jr})]] - \sum_{r \in N_j} G_{ht}^{jr} = 0 \\ , j \in J, t \in T_h, h = 1, \dots, 7, \quad (67) \end{aligned}$$

Please replace constraint (68-74) on page 40 with the following amended constraint (68-74)
 (" , $j \in J'$ is added):

$$X_{ki}^j - Y_{kih}^j - Y_{ki(h-1)}^j - \sum_{m \neq h, (h-1)} Z_{kikh}^j = \sum_{t \in B1kjh} U_{kith}^j \quad , j \in J, i \in I_k, k \in K_j, h = 1, \dots, 7, \quad (68)$$

$$X_{ki}^j - Y_{kih}^j - Y_{ki(h-1)}^j - \sum_{m \neq h, (h-1)} Z_{kikh}^j = \sum_{t \in B2kjh} W_{kith}^j \quad , j \in J, i \in I_k, k \in K_j, h = 1, \dots, 7, \quad (69)$$

$$X_{ki}^j - Y_{kih}^j - Y_{ki(h-1)}^j - \sum_{m \neq h, (h-1)} Z_{kikh}^j = \sum_{t \in B3kjh} V_{kith}^j \quad , j \in J, i \in I_k, k \in K_j, h = 1, \dots, 7, \quad (70)$$

$$r_k Y_{kih}^j = \sum_{l \neq h, (h+1)} Z_{kikh}^j \quad , j \in J, i \in I_k, k \in K_j, h = 1, \dots, 7, \quad (71)$$

$$Z_{kikh}^j \leq Y_{kih}^j \quad , j \in J, i \in I_k, k \in K_j, h = 1, \dots, 7, \\ m \neq h, (h+1) \quad (72)$$

$$X_{ki}^j = \sum_h Y_{kih}^j \quad , j \in J, i \in I_k, k \in K_j, \quad (73)$$

$$QX_{knhi}^j = \sum_{t \in QB1knh} QU_{kniht}^j \quad , j \in J, n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \quad (74)$$

Please replace constraint (75-77) on page 41 with the following amended constraint (75-77)
 (" , $j \in J'$ is added):

$$QX_{knhi}^j = \sum_{t \in QB2knh} QW_{kniht}^j \quad , j \in J, n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \quad (75)$$

$$QX_{knhi}^j = \sum_{t \in QB3knh} QV_{kniht}^j \quad , j \in J, n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \quad (76)$$

$$\sum_{l \in QL_k} A_{klh} Q_{kl}^j = \sum_{n \in F_k} \sum_{i \in I_k} QX_{knhi}^j \quad , j \in J, k \in QK_j, h = 1, \dots, 7, \quad (77)$$

Please replace constraint (80) starting on page 41 with the following amended constraint (80)
("n" added between k and i):

$X_{ki}^j, Y_{kih}^j, Z_{kihm}^j, U_{kiht}^j, W_{kiht}^j, V_{kiht}^j, QX_{knhi}^j, Q_{kl}^j, QU_{knht}^j, QW_{knht}^j, QV_{knht}^j \geq 0$ and
integer for all j, k, n, i, h, and t, and $G_{ht}^{jr} \geq 0$ for all j, r, h, and t. (80)

Please replace lines 18-22 on page 42 with the following amended lines:

G_{ht}^{jr} in formulation (65-80) above doesn't have to be integer even though it represents the number of agents from group j allocated to serve type r contacts. This is due to the fact that ACD's may blend contact types to have an agent serve multiple contact types in a planning period. Hence, formulation (65-80) is a Mixed Integer Programming (MILP) formulation involving both integer and continuous variables.

Please replace lines 9-20 on page 42 with the following amended lines:

The present invention also extends the MILP model to obtain a second formulation by defining pseudo tours for the tours requiring consistent daily shift start times as follows. Consider a tour type, say tour k, $k \in K_j$, available for agent skill skills group j. The set Set K_j contains the valid tour types for agent skill skills group j that require consistent daily shift start times. Constraints (68-73), (78), and six terms ~~in on~~ on the right side of (62) involving variables $X_{ki}^j, Y_{kih}^j, Z_{kihm}^j, U_{kiht}^j, W_{kiht}^j$, and V_{kiht}^j specify the break placement and days off scheduling requirements and agent availability for tours in K_j . The present invention extends the MILP model for the second formulation by first considering all valid ~~days-off work~~ patterns given in FIG. 3 and identifying the patterns satisfying constraints (71-73) for $k \in K_j$. These constraints thus become redundant (i.e. ~~not included~~). A distinct pseudo tour is then defined for every daily start time specified in I_k ,

$k \in K_j$, with the valid work patterns identified. Let one such pseudo tour be α . Then, α has only one daily start time in OI_α representing one of the daily start times in I_k , has the

Please replace objective function (81) starting on page 43 with the following amended objective function (81) (" $\sum_{j \in J}$ " is added):

$$\text{Minimize } \sum_{j \in J} \sum_{i \in QI_k} \sum_{k \in QK_j} C_{kl}^j Q_{kl}^j + \sum_{j \in J} \sum_{k \in QK_j} \sum_{n \in F_k} \sum_h \sum_{i \in QI_k} c_{kni}^j QX_{knh}^j \quad (81)$$

Please replace constraint (83) on page 43 with the following amended constraint (83) (with "+" in front of the 2nd term replaced by "-"):

$$f_{ht}^j(QX, Q, QU, QW, QV) [[+ \sum_{r \in N_j} G_{ht}^{jr}]] - \sum_{r \in N_j} G_{ht}^{jr} = 0 \quad , j \in J, t \in T_h, h = 1, \dots, 7, \quad (83)$$

Please replace constraint (84-87) on page 43 with the following amended constraint (84-87) (" $, j \in J$ " is added):

$$QX_{knh}^j = \sum_{t \in QB1knih} QU_{kniht}^j \quad , j \in J, n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \quad (84)$$

$$QX_{knh}^j = \sum_{t \in QB2knih} QW_{kniht}^j \quad , j \in J, n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \quad (85)$$

$$QX_{knh}^j = \sum_{t \in QB3knih} QV_{kniht}^j \quad , j \in J, n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \quad (86)$$

$$\sum_{l \in QL_k} A_{klh} Q_{kl}^j = \sum_{n \in F_k} \sum_{i \in I_k} QX_{knh}^j \quad , j \in J, k \in QK_j, h = 1, \dots, 7, \quad (87)$$

Please replace constraint (90) starting on page 43 and continuing on to page 44 with the following amended constraint (90):

$$[[\dot{f}_{ht}(X, U, W, V, Y, Z, QX, Q, QU, QW, QV)] \quad \dot{f}_{ht}(QX, Q, QU, QW, QV) =$$

$$\begin{aligned} & \sum_{k \in QK_j} \sum_{n \in F_k} \sum_{i \in QI_k} a_{knih} QX^j_{knih} \\ & - \sum_{k \in QK_j} \sum_{n \in F_k} \sum_{i \in QT1knih} QU^j_{knih} \\ & - \sum_{k \in QK_j} \sum_{n \in F_k} \sum_{i \in QT2knih} (QW^j_{knih} + QW^j_{knih(t-1)}) \\ & - \sum_{k \in QK_j} \sum_{n \in F_k} \sum_{i \in QT3knih} QV^j_{knih} \\ , j \in J, t \in T_h, h = 1, \dots, 7, \quad (90) \end{aligned}$$

Please replace paragraph 1 (lines 9-20) on page 44 with the following amended paragraph:

Extension of formulations (65-80) and (81-89) to the cases discussed in relation to formulations (1-8) and (34-41) is straightforward. It should be clear to those skilled in the art that the information disclosed in relation to formulations (1-8) and (34-41) can be readily utilized as a basis to extend formulations (65-80) and (81-89) of the present invention to the cases involving (i) minimizing scheduled/paid time or maximizing agent preference, (ii) schedule generation when there aren't enough agents (i.e. there are agent shortages in some periods for some contact types), (iii) tours with longer and/or more/less breaks, (iv) fixed days-off, and (v) days with a closure time earlier than the other days. Accordingly, it is to be understood that the embodiments of the invention herein described are merely illustrative of the application of the principles of the invention. Reference herein to details of the illustrated embodiments are not intended to limit the scope of the claims, which themselves recite those features regarded as essential to the invention.

Please replace lines 27-30 on page 44 with the following amended lines:

Formulations (65-80), and (81-89), and their extensions disclosed before can be solved to obtain a global optimal solution in a number of ways. Such a global optimal solution satisfies the necessary and sufficient conditions for optimality of the MILP model of the invention. The solution method of the invention will be discussed in

Please replace the 2nd paragraph on page 45 with the following amended paragraph:

The branch and cut (B&C) algorithm is a well known technique for optimally solving MILP formulations such as (65-80), and (81-89) (Wolsey, 1998). A computer program implementing the B&C algorithm can be developed using any programming language or can be obtained from a number of software companies including Ilog, Inc., and Dash Optimization. These providers are also offering their B&C algorithm in the form of a callable library that can be called from an integrated MILP model generate generator and optimizer module. In either case, the B&C algorithm forms a part of the solution method of the present invention.

Please replace lines 17-31 on page 46 with the following amended lines:

The B&C algorithm first solves the continuous version (i.e. Linear Programming Programming (LP) relaxation) of an MILP model (Wolsey, 1998) (step (17)). This node is called node zero (subsequent nodes are numbered sequentially). If the continuous solution thus found is integer feasible and optimal (step (20)), then the optimizer terminates with an optimal solution to the MILP model (step (21)). If this solution is not integer feasible (i.e. if some integer restricted variables have non-integer values), then the B&C algorithm starts creating nodes (sub-problems) by adding constraints on certain decision variables to eliminate non-integer values. In this case, the objective value of the solution of the LP relaxation is a lower bound on the lowest objective value that can ever be achieved for the scheduling environment formulated in the MILP model. That is, no

solution to the MILP model can have an objective value better than the lower bound thus obtained. ~~Before solving the LP relaxation at node zero, the lower bound is positive infinity if no solution is known beforehand.~~ After solving the LP relaxation at node 0, if the solution is a feasible solution to the LP relaxation but violates some integrality constraints (step (22)), the B&C algorithm calls the RA algorithm. If the RA algorithm locates an integer feasible

Please replace step 31 on page 48 and continuing on to page 49 with the following amended step:

Step 31: Compute the scheduled agent availability in each period for all agent groups (i.e. $f_{ht}^j(X, U, W, V, QX, Y, Q, QU, QW, QV)$ in equation (67)). Compare the number of agents available in each agent group j in a planning period, say $t \in T_h$ on day h , with the values of the agent allocation variables G associated with this agent group in planning period t . If the number of agents available to work for agent group j is greater than not equal to the sum of the current values of the agent allocation variables (from step 25) for all contact types in N_j in period t on day h , allocate the excess agents adjust values of associated allocation variables to meet the requirements by contact types in N_j . Prioritize contact types in each period by (i) maximum shortage, (ii) whether this agent

Please replace step 34 and 35 on page 49 with the following amended step:

Step 34: If a feasible schedule satisfying all agent requirements is found either by the B&C algorithm, or in the earlier calls to the RA algorithm, go to step 35. Otherwise, go to step 36.

Step 35: Discard the current infeasible solution (that is, an integer solution with agent shortages). Stop the RA algorithm, and return control back to the B&C algorithm (step (17)).

Please replace step 36 on page 46 with the following amended step:

Step 36: If an integer feasible solution to the MILP model hasn't been found by the B&C and RA Algorithms at earlier tries, compare the new infeasible solution with the infeasible solution recorded in earlier calls to the RA algorithm, and update the best infeasible solution if the new one has fewer period shortages. If this is node zero, store the infeasible solution if there are no known solution. ~~Otherwise, stop Stop~~ the RA algorithm, and return back to the B&C algorithm (step (17)).

Please replace step 37a on page 50 with the following amended step:

37a: For each agent skill group j with available agents (i.e. the number of agents already scheduled in the rounded solution found in step (25) is less than the maximum number of agents available for each an agent skill group), consider tour type k that is available for this agent group. If the tour requires consistent daily start time and shift length, that is $k \in K_j$, go to
37b. Otherwise, $k \notin QK_j$ and go to **37c.**

Please replace step 37e on page 51 and continuing on to page 52 with the following amended step:

37e: If tour k requires consistent daily shift start times and lengths, compare the peak shortage covered by tours starting at different times in P_k , $k \in K_j$, and find the start time i^* with the highest peak shortage covered. If there are ties between two or more start times in P_k , select the one with the highest shortage coverage per dollar

$$(\text{total shortage coverage}) / C_{ki}^j$$

If tour k does not require consistent daily shift start times and lengths, compare the peak shortage covered by tours with different work patterns in QL_{kj} , and $k \in QK_j$, and find the work pattern l^* with the highest peak covered. If there are ties between two or more work patterns in QL_{kj} , $k \in QK_j$, select the one with the highest shortage coverage per dollar

$$\frac{(\text{total shortage coverage})}{(C_{kl}^j + \sum_h (\text{number of hours worked on day } h) * c_{kni}^j)}$$

Please replace step 37f on page 52 and continuing on to 53 with the following amended step:

37f: If tour k for agent group j is the first tour evaluated, then store the current agent skills group, tour schedule, and agent allocation as the best ones found so far, and go to step 37a. Otherwise, if the peak shortage covered by tour k is higher than the highest peak shortage covered by any tour and agent skills group considered so far, update the best agent skills group, tour schedule, and agent allocation with the current ones. If there is a tie between the current tour schedule and agent skills group, and the best agent skills group and tour schedule found so far, select the one with the highest

$$\frac{(\text{total shortage coverage})}{C_{\underline{x}_{ki}}^j}, \text{ for } i \text{ when } k \in K_j,$$

$$\frac{(\text{total shortage coverage})}{(C_{kl}^j + \sum_h (\text{number of hours worked on day } h) * c_{kni}^j)}, \text{ for } l \text{ when } k \in QK_j,$$

Also update the peak shortage covered, and the total shortage coverage for the current tour schedule. Go to 37g if all agent skills groups and all tours are evaluated. Otherwise, go to 37a.

Please replace step 37g starting on page 53 and continuing on to page 54 with the following amended step:

37g: Add the best tour schedule found in steps **37a-37f** to the schedule developed in steps 16-37. Update the values of shift, days-off, break, and agent allocation X, Y, U, W, V, QX, Q, QU, QW, QV, and G variables. Update shortages and excesses using the expanded schedule with the new agent.

Please replace step 39 starting on page 53 and continuing on to page 54 with the following amended step:

Step 39: Consider the agents scheduled in the expanded schedule. Start with the most expensive agent skills group and consider the tours scheduled for this group. If there is a tour that can be removed without creating any shortages in any period and contact types, the current solution has redundant tours. Consider all agent group skills groups in this fashion to determine redundant agents. Go to step **40** to remove the redundant tours. Otherwise, go to step **41**.

Please replace paragraph 1 on page 54 with the following amended paragraph:

The RA algorithm can also be supplemented with a two-way interchange, or general n-way interchange step so that, when an integer feasible solution is found, the RA algorithm can make an attempt to improve it by replacing one or more scheduled tours with other tours that are not currently scheduled. This is a well-known enhancement in the field of optimization that is widely applied to a number of other optimization

problems. Also, if chosen, the solution method of the invention may also be terminated prior to finding an optimal solution if, for example, (i) the objective function for the best known integer solution differs less than a pre-specified percentage from the best lower bound found by the B&C algorithm, (ii) a pre-specified number of sub-problems (e.g. one or more sub-problems) are solved by the B&C and RA algorithms, or (iii) a pre-specified amount of time elapses after locating the first integer feasible solution to the MILP. A solution that is either optimal or satisfying these conditions is referred to as a “terminal” solution. The RA algorithm may also be modified to search for an integer feasible solution to a sub-problem using any of the known heuristics (e.g. Tabu Search, Simulated Annealing, Genetic Search), or to formulate and add Gomory’s cuts or other well known cuts (Wolsey, 1998) to sub-problems, change branching direction or priorities in the B&C algorithm. The optimization process outlined here doesn’t exclude these extensions.

Please replace paragraph 2 on page 54 with the following amended paragraph:

Once a terminal solution is found, detailed tour schedules for agents can be developed in a number of ways. All such schedules will have the same objective value and, therefore, equally effective. First, work ~~and non-work pattern~~ patterns and daily shift start times are specified. Consider a tour requiring consistent daily start ~~time~~ times. The days-off scheduled in the solution for this tour are first assigned to individual tours. For example, if, in the terminal solution, tour type 1 has 2 agents from skill group 3 to start daily at 7:00am (that is, the variable $X^3_{1(7:00)} = 2$ in formulation (65-80)), initially these 2 agents

Please replace lines 9-11 on page 55 with the following amended lines:

in the terminal solution by the values of the decision variables are assigned to work patterns specified by the values of the work pattern variables in the terminal solution. Work pattern constraints (e.g. constraint (77)) ensure that a daily shift is scheduled for